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13. ABSTRACT (Maximum 200 words)

This is the final report for the work on information and aggregation for cooperative systems. We describe the objective of the work and the major accomplishments of this research.

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Enclosure 1

FINAL REPORT
Objective

The problem of the fusion of information provided by multiple sources appears in many applications. It plays a central role in the task of situation awareness. It is required to enable cooperation and coordination in military units. In addition to information about observations and perceptions about a particular situation we need consider information about criteria and goals of various members of the organization. Information provided by human sources are often expressed in linguistic terms which are inherently imprecise. The overall objective of this research is the development of tools to aid in this complex task of information fusion. In support of this objective we look at a number of related problems. Among these problems are the development of structures for the representation of information and the establishment of mathematical aggregation operators to allow the fusion of information.

Approach

The problem of information fusion can generically be seen to involve the following basic steps

1. Collect information from various sources

2. Translate the information into some formal mathematical representation

3. Intelligently combine the information with the aid of mathematical aggregation operators

4. Retranslate the mathematical structures that result from the fusion in step three into a language appropriate to be presented to human clients

While step one is clearly an important part of the process our main focus is on steps 2-4. Because of the inherent imprecision and uncertainty involved in much of the battlefield type information considerable use will be made of modern technologies that allow the formal mathematical representation this kind of information. In particular we make use of the frameworks provided by the theory of fuzzy sets and the Dempster-Shafer mathematical theory of evidence.

Scientific Barriers

Often the information available to military decision makers is based upon human perceptions rather then precise measurements. This is particularly the case with respect to information about an adversaries capabilities or intentions. Because of the human limitations in

resolving detail, storing information and manipulating observations these perceptions are generally imprecise (granular). Statements such as *"The enemy appears to have substantially reduced the intensity of its defense"* or *"The probability that their fuel supply will last more than a couple hours is small"* or *"The local population seems to growing tired of the disruption caused by the resistance fighters"* are examples of this type of perception based information. Structures are needed for the representation of this type of information.

Significance

We believe that the work we are doing will help in the development of more intelligent human centered military decision support systems. This will be accomplished by allowing for the formal inclusion in these systems of the types of imprecise information described above. In addition the technologies being developed will allow for the representation of the complex concepts needed to capture the goals and criteria of interacting agents

Major Accomplishments

Group Negotiation Framework

Negotiations between participants is an important part of the process of coordination and cooperation. Since future military units will more and more consist of a combination of humans, robot soldiers and other non-human autonomous agents there is a need for the development of tools and formal mathematical concepts to enable the cooperation and coordination between these various possibly heterogeneous components. We provided a framework for automated multi-agent negotiation. This framework involves a mediation step in which the individual agent preference functions are aggregated to obtain a group preference function. It involves a selection procedure for choosing an alternative based on the final group preference function. Considerable attention was focused on the implementation of the mediation rule where we allowed for a linguistic description of the rule using fuzzy logic. A particularly notable feature of our approach is the inclusion in the mediation step of a mechanism rewarding the agents for being open to alternatives other then simply their most preferred.

Prioritized Aggregation Operators

Many decision processes require a prioritization of goals and criteria. For example in planning operations the safety of soldiers has highest priority. A chain of command structure imposes prioritization of goals and objectives We developed formal mathematical aggregation operators that allows decision making with prioritized criteria. Our approach assigns

importance to lower priority criteria dependent on its satisfaction of higher priority ones

Technology Transfer

In cooperation with with researchers from the NASA Jet Propulsion Lab and U. of Calgary we developed a framework for a **biometric based screening decision support system** that can be integrated within physical access control systems such as airport or border checkpoints to aid the security personnel. Central to our approach is a novel combination of using biometric sensor data with soft computing reasoning and inference techniques developed with our ARO support. As individuals queue and pass through these access points biometric sensed data using video, infrared and audio sensors is used to capture information about features of appearance (both natural, such as aging and intentional, such as surgical changes), physiological characteristics (temperature, blood flow rate), and behavioral features (voice and gate). This information is inputted to the second component of system, our knowledge base. Using techniques from computational intelligence such as fuzzy logic, this information is intelligently processed in a human-like fashion for indications of atypical physical features and/or aroused (not ordinary) emotional states. The output of this will be passed to a screen of the access security personnel.

With the overwhelming daily flow of data and information intelligence analyst need approaches to assist in focusing this flow to help detect any impending critical situations such as domestic or foreign terrorism. In cooperation with researchers at Stennis Space Center we have began applying techniques developed with our ARO support to the development of a system to aid in this task. Central to our approach is the use of multiple concept hierarchies to generalize the input data and uncover common concepts that may be found in the data. This generalized input is feed to our knowledge base to make further inferences about the current situation. The generalized inputs along any pertinent inferences are feed to a component which instantiates potentially critical situations. These situations are brought to the attention of an analyst for further investigation.

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Multi-Agent Negotiation Using Linguistically Expressed Mediation Rules

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Abstract

The problem of multi-agent negotiation is considered. We provide an framework for the multi-agent negotiation process in which each of the participating agents provides a preference function over the set of alternatives. This framework involves a mediation step in which the individual agent preference functions are aggregated to obtain a group preference function. The determination of the satisfaction of a stopping rule which decides whether a suitable final group preference function has been obtained or whether the agents must participate in another round of mediation. It also involves a selection procedure for choosing a alternative based on the final group preference function. We describe various implementations for these different steps. Considerable interest is focused on the implementation of the mediation rule where we allow for a linguistic description of the rule using fuzzy logic. A particularly notable feature of our approach is the inclusion in the mediation step of a mechanism rewarding the agents for being open to alternatives other then simply their most preferred.

Key words: mediation, OWA operators, preference aggregation, agents, fuzzy sets

1. Introduction

The use of agents on the internet will be an important part of the future internet culture (Jennings and Wooldridge 1998; Wooldridge 2002; Parsons, Gmytrasiewicz, and Wooldridge 2002). These agents will play a major role in performing personal tasks for their human masters as will as being central to many aspects of E-Commerce. This situation has led to considerable interest in the development of agent technology and in particular interaction between agents. One form of interaction between agents is negotiation (Faratin, Sierra and Jennings 1997; Beer etc 1999; Bartolini, Preist, and Jennings 2005; Rosenschein and Zlotkin 1994; Kraus, 2001; Yager 1997).

The process of providing for automated multiple agent negotiation can be seen to require among other things addressing at least the following two major tasks. The first task is the construction of the protocol or framework to be used to implement the negotiating process. This essentially involves describing the rules under which the negotiation process will take place.

The second task, given the protocol, involves the investigation of the appropriate strategies an agent uses in trying to optimize their reward (satisfaction/payoff). In this work we shall begin to look at possible protocols for implementing multi-agent negotiations. While we shall make some comment on the task of determining strategies we leave this for a future work. We note that the task of determining an agent's best strategy is often extremely complex.

2. Basic Protocol for Multi-Agent Negotiation

In the following we begin to describe a framework for the multi-agent negotiation We shall assume a collection of n agents and a set of X of alternatives. One of these alternatives must be selected as the group choice. Furthermore, we assume that each agent has a real or virtual mapping[1] V_i that associates with each alternative x_j a value $V_i(x_j) = v_{ij}$ indicating his perceived payoff for the situation in which alternative x_j is chosen by the group. An agents objective is to maximize his payoff. The agent's perceived payoff need not be made available to other agents in the negotiation although we don't preclude the possibility of one agent learning some information about other agent's perceived worth of the alternatives. This issue will be of more significance when one considers the task of having agents develop optimal negotiation strategies.

The basic protocol of our proposed negotiation process is the following:

1. Each agent provides a preference function in terms of a mapping $A_j: X \rightarrow [0, 1]$ such that $A_j(x_i)$ indicates agent j's support for alternative x_i.
2. The individual agent preference functions are aggregated to obtain a group preference function. We shall refer to this as the **mediation** step.
3. A stopping rule is tested

  (i) If the conditions of the stopping rule is meet we go to step 4.
 (ii) If the conditions of the stopping rule is not met we go to step 5.

4. Select an alternative based in the current group preference function and the negotiation process is terminated.
5. Provide each of the agents with some information about the preceding mediation process, ie: resulting group preference function.
6. Go to step one.

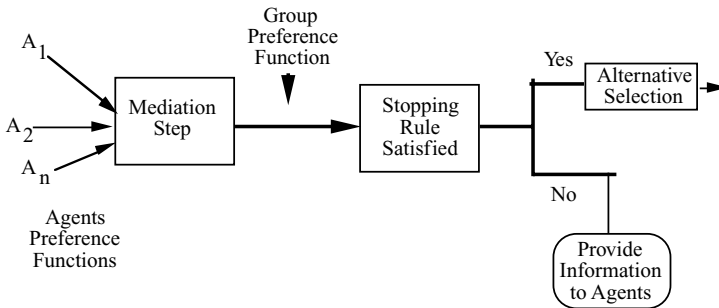In Figure 1 we illustrate this algorithm:

Figure 1. Negotiation protocol.

We assume that the negotiation process is such that a solution from the set X is always obtained. This means that we are not considering situations in which we are reformulating the solution set. The set of possible solutions is defined "a priori" and is fixed. This also assumes the agents can't withdraw. We also assume that the stopping rule is always satisfied in a finite number of steps.

Essentially, the multi-agent negotiation is a dynamic process where at each stage of the process an agent provides a preference function determined by its underlying payoff function and any information available about the previous stages of the negotiation.

As we shall subsequently see it is the process of choosing these preference functions at each round of the negotiations then constitutes a participating agent's strategy. An important consideration in an agent's determination of their strategy are the rules and procedures used in the negotiation process, steps 2–5 in the preceding negotiation algorithm. Here we now turn to describing the protocol used in steps 2 to 5 of the negotiation process. We shall describe various possible implementation of these steps.

3. The Mediation Step

Here we look at the mediation step (Yager 2004). This involves the process aggregating the individual preference functions of the participating agents. It must be pointed out this is a process taking place at each round in the negotiation process.

Our point of departure here is a collection of n agents and a set X of alternatives. We assume each agent has provided a preference function A_i over the set X such that for $A_i(x)$ indicates the degree to which agent A supports alternative x. We can view A_i as a fuzzy subset over X. Our objective in this mediation step is to obtain a group preference function D: $X \rightarrow [0, 1]$ which associates with each alternative $x \in X$ a value we $D(x) = F(A_1(x), \ldots, A_n(x))$ indicating the degree to which x is supported by the group of agents.[2] The form of F is called the mediation rule, it describes the process of combining the individual preferences. Our goal here is to investigate different possibilities for mediation rules. We note that we can give the mediation process an anthropomorphic nature and refer to it as a mediation agent or simply a mediator.

One desired feature of any mediation rule is what Arrow (1951) calls positive association between the group and the individual preferences. This essentially requires that if any agent's preference for some alternative increase the group's preference for that alternative should not decrease. More formally this requires that F be monotonic. That is, if x and y are two alternatives and if of $A_j(x) \geq A_j(y)$ for all j then $D(x) \geq D(y)$. Another requirement we desire in the mediator F is fairness. This means that all of the participating agents are treated in the same way. This is closely related to what mathematicians call commutativity (symmetry).

Beyond satisfying these properties the choice of the form for F can be used to reflect a desired mediation imperative for aggregating the preferences of the individual agents to get the group preference function.

Starting with the work of Bellman and Zadeh fuzzy logic has provided tools to aggregate preference functions. In Bellman and Zadeh (1970) the authors suggested using as an

aggregation imperative a desire to satisfy **all** the agents. Using this the authors obtained a representation of D as

$$D \equiv A_1 \text{ and } A_2 \text{ and } \ldots \text{ and } A_n$$

This leads to a formal implementation as

$$D(x) = \text{Min}_j[A_j(x)]$$

Subsequent understanding of the nature of the multivalued logical "and" led researchers to provide alternate definitions for the operator "and" using a t-norm (Klement, Mesiar and Pap 2000). This provides a more generalized class of "anding" operators in addition to the Min. In addition to the Min another example of t-norm is the product, $D(x) = \prod_{j=1}^{n} A_j(x)$. We note that the use of the product operator results in a mediation rule closely related to that used in the Nash (1950) bargaining model.

As noted by Yager (1988) the imperative of requiring that "all agents" be satisfied by a solution may not be suitable for multi-agent preference aggregation. Is is a very strong requirement that all agents be satisfied. In particular, we see that a single agent can unilaterally doom any criteria, if $A_j(x) = 0$ then $D(x) = 0$. In addition to giving each agent a lot of power it is a difficult imperative to satisfy. Essentially this "anding" type aggregation gives the most influence to the agent with the lowest score for the alternative.

Other mediation rules can be considered where the support of all the agents is not needed and thereby relaxing this unilateral control. For example, a solution may be acceptable if **_most_** of the agents support it. In an attempt to provide a more general class of aggregation rules Yager (1996) introduced the idea of quantifier guided aggregation. This approach allows a natural language expression of the quantity of agents that need to agree on an acceptable solution. As we shall see the Ordered Weighted Averaging (OWA) operator (Yager 1988; Yager and Kacprzyk 1997). will provide a tool to model this kind of softer mediation rule.

4. OWA Operators

In order to be able to formally model these quantifier guided aggregations we shall use the class of aggregation operators called **O**rdered **W**eighted **A**veraging (OWA) operators introduced by Yager (1988).

Definition: An aggregation operator $F : I^n \rightarrow I(I = [0, 1])$ is called **O**rdered **W**eighted **A**veraging (OWA) operator of dimension n if it has an associated weighting vector $W = [w_1 w_2 \ldots w_n]$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$ and where $F(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j$ where b_j is the jth largest element of the aggregates $\{a_1, \ldots, a_n\}$

An essential feature of this aggregation is the reordering operation, which is a nonlinear operation. In the OWA aggregation the weights are not directly associated with a particular argument but with the ordered position of the arguments. If ind is an index function such that $\text{ind}(j)$ is the index of the jth largest if the arguments than we can express $F(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j a_{\text{ind}(j)}$.

Yager (1988) shows that OWA aggregation has the following properties:

(1) **Commutativity**: The indexing of the arguments is irrelevant
(2) **Monotonicity**: If $a_i \geq \hat{a}_i$ for all i then $F(a_1, \ldots, a_n) \geq F(\hat{a}_1, , \hat{a}_n)$.
(3) **Idempotency**: $F(a, \ldots, a) = a$.
(4) **Boundedness**: $\text{Max}_i[a_i] \geq F(a_1 \ldots \ldots, a_n) \geq \text{Min}_i[a_i]$

We note that these conditions imply that the OWA operator is a mean operator.

With the OWA operator the form of the aggregation is dependent upon the associated weighting vector. Yager (1993) investigated various different families of OWA aggregation operators. A number of special cases of weighting vector are worth noting. The vector W^* defined such that $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ gives us the aggregation $F^*(a_1, \ldots, a_n) = \text{Max}_i[a_i]$. Thus W^* provides the largest possible aggregation. The vector W_* defined such that $w_n = 1$ and $w_i = 0$ for $i \neq n$ gives us the aggregation $F^*(a_1, \ldots, a_n) = \text{Min}_i[a_i]$. This vector provides the smallest aggregation of the arguments. The weighting vector W_{ave} defined such that $w_i = 1/n$ for all i gives us the simple average $F_A(a_1, \ldots, a_n) = \frac{1}{n} \sum_{i=1}^{n} a_i$. The weighting vector W^k defined such that $w_k = 1$ and $w_i = 0$ for $i \neq k$ gives us $F(a_1, \ldots, a_n) = b_k$ where b_k is the kth largest of the a_i. Another case of OWA aggregation is called the Olympic operator, in this case $w_1 = w_n = 0$ and $w_i = \frac{1}{n-2}$ for the other components. In this case we are eliminating the extreme scores.

Associated with any OWA aggregation a measure called its attitudinal character. If W is a weighting vector of dimension n then the attitudinal character is defined as

$$A - C(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i.$$

It is easy to show that this measure lies in the unit interval. Furthermore it can be shown that $A - C(W^*) = 1$, $A - C(W_{ave}) = 0.5$ and $A - C(W_*) = 0$. In the framework of multi-agent preference aggregation the attitudinal character can be seen as being inversely related to an individual's power in rejecting an alternative. Thus $A - C(W) = 0$ indicates that any individual agent can unilaterally doom an alternative. As $A - C(W)$ moves from zero to one the individual agents power of rejection decreases, more agreement is needed to doom an alternative.

An interesting family of OWA operators are the $E - Z$ OWA operators. Actually there are two families. The first family is defined via a weighting vector W such that

$$w_i = 1/q \quad \text{for} \quad i = 1 \text{ to } q$$
$$w_i = 0 \quad \text{for} \quad i = q + 1 \text{ to } n$$

Here the first q weights are the same and the remaining weights are zero. For this operator $A - C(W) = \frac{1}{2} + \frac{1}{2}(\frac{n-q}{n-1})$. In particular $A - C(W) \geq 0.5$. Here we are taking the average of the q largest arguments.

The other family of $E - Z$ operators are such that W is defined by

$$w_i = 0 \qquad \text{for} \quad i = 1 \text{ to } q - 1$$
$$w_i = \frac{1}{n-q} \quad \text{for} \quad i = q + 1 \text{ to } n$$

Here the last $n - q$ weights are the same and the first q weights are zero. We see in this case that $A - C(W) = \frac{1}{2}(\frac{n-q}{n-1})$. and therefore $A - C(W) \leq 0.5$. We note that as we increase the number of elements that have zero weight, make q larger, we decrease the value of $A - C(W)$. When all elements except the last, w_n, are equal to zero we obtain the Min operator. We see that this operator can provide a softening the original Min mediation rule.

5. Quantified Guided Mediation

In the preceding when we used the Min for calculating our group preference function we have essentially implemented the following linguistic agenda for our mediation rule.

   All *agents must be satisfied by a solution.*

As we noted in many situations the requirement that ***all*** agents be satisfied is too strong and more reasonable mediation rules might be.

   Most *agents must be satisfied by a solution.*
   At least about half the *agents must be satisfied by a solution.*

The above statements are examples of what we call **quantifier guided aggregations**. In these statements the underlined terms are examples of what Zadeh (1983) called relative linguistic quantifiers.

In natural language we find many examples of relative linguistic quantifiers. These objects are exemplified by terms such as *all, most, many, at least half, some* and *few*. Zadeh (1983) suggested a formal representation of these linguistic quantifiers using fuzzy sets. He suggested that any relative linguistic quantifier can be expressed as a fuzzy subset Q of the unit interval $I = [0, 1]$. In this representation for any proportion $y \in I$, $Q(y)$ indicates the degree to which y satisfies the concept expressed by the term Q. In most applications of the quantifier guided aggregation we use a special class of these linguistic quantifiers, called **R**egular **I**ncreasing **M**onotone (RIM) quantifiers. These types of quantifiers have the property that as more agents are satisfied our overall satisfaction can't decrease. Formally, these quantifiers are characterized in the following way:

   1. $Q(0) = 0$, **2**. $Q(1) = 1$ and   **3**. $Q(x) \geq Q(y) \, if \, x > y.$

Examples of this kind of quantifier are *all, most, many, at least α*.

The quantifier **for all** is represented by the fuzzy subset Q_* where $Q_*(1) = 1$ and $Q_*(x) = 0$ for all $x \neq 1$. The quantifier **any** is defined as $Q^*(0) = 0$ and $Q^*(x) = 1$ for all $x \neq 0$. Both of them are examples of RIM quantifiers.

Having introduced the OWA aggregation operator we are now in a position to describe the process of quantifier guided aggregation. Again assume that we have a collection of n agents. These agents have their preferences represented as fuzzy subsets over the set of alternatives X, $A_1, A_2, \ldots, A_n$. Under the quantifier guided mediation approach a group mediation protocol is expressed in terms of a linguistic quantifier Q indicating the proportion of agents whose agreement is necessary for a solution to be acceptable. The basic form of the mediation rule in this approach is

Q agents must be satisfied by an acceptable solution,

where Q is a quantifier.

The formal procedure used to implement this mediation rule is described in the following. The quantifier Q is used to generate an OWA weighting vector W of dimension n. This weighting vector is then used in an OWA aggregation to determine the group support for each alternative. For each alternative the argument of this OWA aggregation is the degree of support for that alternative by each of the agents, $A_i(x), i = 1 \ldots n$. Thus the process used in quantifier guided aggregation is as follows:

(1) Use Q to generate a set of OWA weights, $w_1, \ldots, w_n$.
(2) For each alternative x in X calculate the overall group support

$$D(x) = F(A_1(x), A_2(x), \ldots, A_n(x))$$

The procedure (Yager 1996) used for generating the weights from the quantifier is

$$w_i = Q(\frac{1}{n}) - Q(\frac{i-1}{n}) \qquad \text{for } i = 1 \ldots n.$$

Because of the nondecreasing nature of Q it follows that $w_i \geq 0$. Furthermore from the regularity of Q, $Q(1) = 1$ and $Q(0) = 0$, it follows that $\sum_i w_i = 1$. Thus we see that the weights generated are an acceptable class of OWA weights.

The use of a RIM quantifier to guide the aggregation essentially implies that the more agents satisfied the better the solution. This condition seems to be one that is naturally desired in multi-agent mediation. In Figure 2 we show a prototypical example of a RIM linguistic quantifier and illustrate the process of determining the weights from the quantifier. Here we see that $w_j = Q(\frac{j}{1}) - Q(\frac{j-1}{n})$. We see that the weights depend on the number of agents as well as the form of Q.

In Figure 3 we show the functional form for the quantifiers "all" and "any", Q_* and Q^*. As we have indicated in the first case, "all", we get the weighting W_* where $w_n = 1$ and in the second case, "any", we get the weighting W^* where $w_1 = 1$.
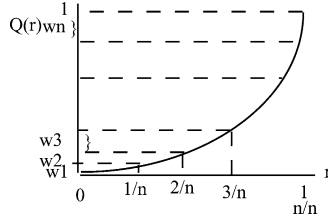
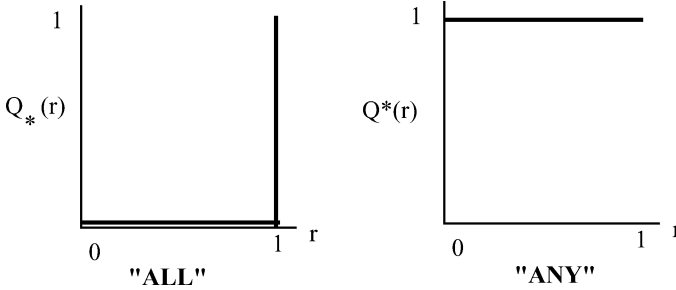*Figure 2.* Getting the weights from the quantifier.

Figure 3. Quantifiers "all" and "any".

One feature which distinguishes the different types of mediation rules is the power of an individual agent to eliminate an alternative. For example in the case of "all" this power is complete. In order to capture this idea we shall associate with each linguistic quantifier Q a measure called the **V**alue **O**f **I**ndividual **D**isapproval (VOID) which we define as

$$\text{VOID}(Q) = 1 - \int_0^1 Q(r)\,dr$$

It is easy to see $\text{VOID}(Q) \in [0, 1]$. We also note that $\text{VOID}(Q_*) = 1$ and $\text{VOID}(Q^*) = 0$. We note that it is an expression of the complement of the attitudinal character in terms of the quantifier function instead of the weighting vector W.

Consider the class of linguistic quantifier that corresponds to "at least αth percent". This corresponds to a mediation imperative that says "at least $100 \times \alpha\%$ of the agents must be satisfied for a solution to be acceptable." This type of quantifier can be represented as $Q(r) = 0$ if $r < \alpha$ and $Q(r) = 1$ if $r > \alpha$ (see Figure 4). For this quantifier $\text{VOID}(Q) = \alpha$. We also note that here $w_j = 1$ for j such that $j \geq \alpha n$ and $j - 1 < \alpha n$. Here all other OWA weights are zero.

In our approach thus far we describe our mediation rule in terms of a RIM quantifier expressed in terms of natural language. We then convert this linguistic descriptions into a fuzzy subset Q on the unit interval. Here Q is a function $Q: [0, 1] \rightarrow [0, 1]$ such that $Q(x) \geq Q(y) : x \geq y$ $Q(1) = 1$ and $Q(0) = 0$. We denote functions with these properties as **B**asic **U**nit-interval **M**onotonic (BUM) functions. The requirement that we initially describe our mediation rule using a natural language verbal description can be
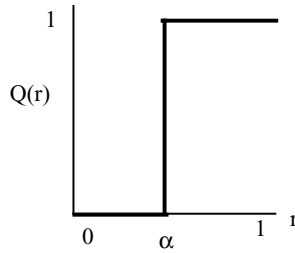
Figure 4. At least α percent.

limiting. More generally any function Q having the properties of a BUM function can be seen to be an appropriate form for generating mediation rules. Thus we can consider two approaches to generating these quantifier based mediation rules. One approach is to start with a linguistic expression and then obtain the associated function Q. A second approach is to allow the mediation rule to be directly expressed in terms of a function Q. One very important characteristic of this second method is that we can easily introduce into our mediation a number of formal properties that are not very easily expressed using a verbal description of the quantifier.

Thus in our approach we shall not feel restricted in our method of describing the mediation rule. We shall feel free to use natural language or formal functions to describe our quantifier. This ability to navigate between language and formal mathematical structures is one of the benefits of using fuzzy set methods.

We now consider some examples of quantifiers which are directly expressed in term of formal function. We note while these result in some very common forms of mediation rules they are not naturally expressible in linguistic terms .

A very important example of quantifier is the linear one, $Q(r) = r$ see Figure 5. For this quantifier we get $wj = \frac{1}{n}$, all the agents get the same weight. Also for this VOID$(Q) = 0.5$.

Closely related to this linear function are two other functional forms. As we shall see these functions give weighting vectors close to the $E - Z$ OWA aggregation (Yager 2003). The first of these Q_Z is shown in Figure 6. For this quantifier VOID$(Q) = \frac{1}{2}(1 - \beta)$ and therefor VOID$(Q) \in [0, 0.5]$.
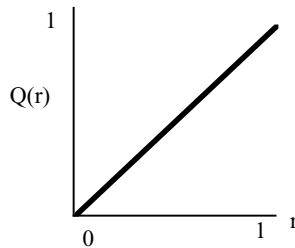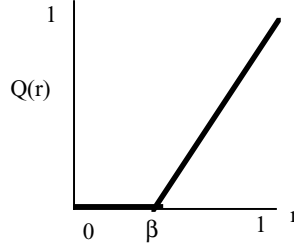

Figure 5. Linear quantifier.

Figure 6. Piecewise linear Q_{Z_β} quantifier.

Let us look at the weights generated by Q_{Z_β}. Here for simplicity we shall assume ßn is an integer. Let us denote this integer as q. In this case with $w_j\, Q(\frac{j}{n}) - Q(\frac{j-1}{n})$ we easily see that $w_j = 0$ for j ≤ q and $w_j = \frac{1}{n-q}$ for $j > q$ The important observation here is that the weights go into two categories: those that are zero and those that are equal. We see this the $E - Z$ OWA weights.

We note that if ßn $= q$ is not an integer then we get one additional weight class that transitions until we get integers. This if q_1 is the largest integer less than q and if q_2 is the smallest integer larger than q. We have

$$
\begin{aligned}
w_j &= 0 & j &\le q_1 \\
w_j &= \frac{q_2 - q}{n(1 - \beta)} & j &= q_2 \\
w_j &= \frac{1}{n - q_2} & q_2 + 1 &\le j \le n
\end{aligned}
$$

Closely related to this quantifier Q_{z_α} shown is Figure 7. In this case VOID(Q_{z_α}) $= 1 - \frac{1}{2}\alpha$ and we see that VOID(Q_{z_α}) $\in [1, 0.5]$.

Between these two functions, Q_{Z_β} and Q_{Z_α}, we can attain any degree of VOIDness. For the most part in multi-agent preference mediation we are interested in the situation in which VOID(Q) ≥ 0.5. That is we desire many of the agents satisfied.
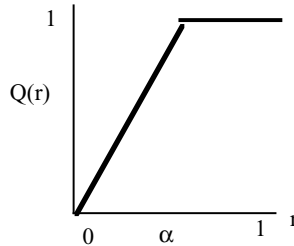

Figure 7. Piecewise linear Q_{Z_α} quantifier.

Another family of quantifiers are those expressed by $Q(r) = r^P$ for $p > 0$. In this case

$$\text{VOID(Q)} = 1 - \int_0^1 r^P \, dr = \frac{1}{p+1} \ .$$

For this quantifier with $w_j = (\frac{j}{n})^p - (\frac{j-1}{n})^p$ we see that as p increases we get closer to the Min. On the other hand as p gets closer to zero we get the Max.

6. Understanding the Negotiation Process

What must be kept in mind in the negotiation process is that each agent has a single interest, the maximizing of its payoff as reflected by the function V_i. We recall $V_i(x)$ is the payoff attained by agent i if the group selects alternative x. For any x the greater the group support for x, $D(x)$, the greater the possibility of x being selected. Hence one focus of the individual agent is to try to get high scores in D for those alternatives he desires and low scores for those alternative he doesn't. With this in mind an agent's choice of preference function at each stage in the negotiation process is his vehicle for attaining this goal. With $D(x) = F(A_1(x), \ldots, A_n(x))$ and the fact that F is monotonic leads to a direct approach for each agent in attaining their goal, give high scores to those you want and low scores to those you don't. Thus the monotonicity of the aggregation process rewards complete selfishness. Support only those alternatives you want.

On the other hand, by its very nature, the process of negotiation is one in which we are trying to accommodate the views and desires of other agents. This suggests that an appropriate negotiation process should reward those participating agents that are more open to other people's views. That is we want to reward those agents who are not totally selfishly only focusing on their own self interest. We need some mechanism in the mediation process to reward agents that are most adaptable with respect to the solutions they can accept. Thus where the monotonicity property of the preference aggregation function process correctly drives the mediation process by the self interest of the individual agents, we need some additional mechanism to reward those agents that are most open to solutions. One reasonable mechanism for introducing this consideration is to increase the importance of those agents who are more accommodating regarding the choice of alternatives. In Figure 8 we illustrate our various considerations in the formulation of the mediation function.

In anticipation of providing a mechanism for rewarding agent openness we turn to the issue of including an agent's importance in the quantifier guided aggregation process.

7. Importance Weighted Quantifier Guided Aggregation

We now turn to the implementation of the quantifier guided multi-agent mediation step in the case in which the agents can have differing importance (Yager 1997). We again assume we have a set of n agents whose preferences are expressed as fuzzy subsets, A_i, over the space of alternative solutions X. We now additionally assume that we can associate with

POSTIVE ASSOCIATION
Consider Individual
Agents Selfish Preferences

⇓

| Individual Agents Preference Functions | → | **MEDIATION** Aggregation of Preference Functions | → | Group Preference Function |

⇑

Reward
Agent
Being
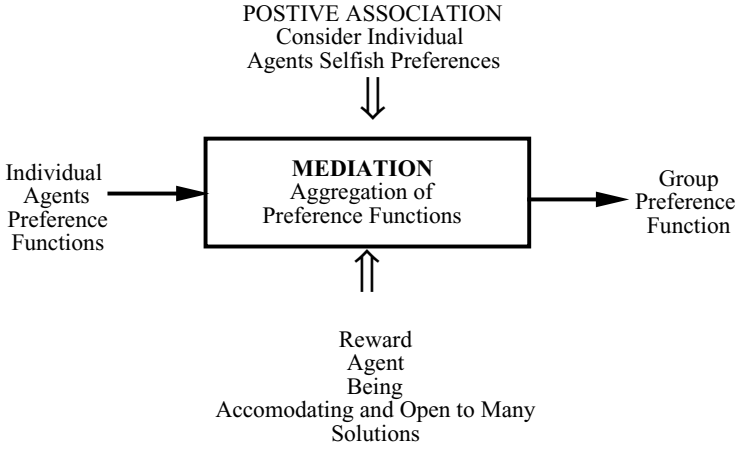Accomodating and Open to Many
Solutions

Figure 8. Considerations in mediation process.

each agent a value t_i indicating the importance of that agent in the mediation process. All we need assume is that the t_i's are non-negative although here we restrict ourselves to the special case where $V_i \in [0,1]$. We make no restrictions on the total value of importances, that is they need not sum to one.

In introducing quantifier guided aggregation we noted that our mediation rule was expressed by the statement *Q agents are satisfied by x*. In this case with importances we modify the mediation rule to be

*Q **important** agents must be satisfied by an acceptable solution.*

In the following we describe the procedure to evaluate the overall support for an alternative x under this rule. First we note that for a given alternative x we have a collection of n pairs $(t_i, A_i(x))$, here t_i is the importance of the ith agent and $A_i(x)$ is the support it gives to alternative x. The first step in this process is to order the $A_i(x)$'s in descending order. We let b_j be the jth largest of $A_i(x)$, $b_j = A_{id(j)}(x)$. Furthermore, we let u_j denote the importance associated with the agent that gives the jth largest support to x, $u_j = t_{id(j)}$. Thus if $A_5(x)$ is the largest of the $A_i(x)$ then $b_1 = A_5(x)$ and $u_1 = t_5$. At this point we can consider our information regarding the alternative x to be a collection of n pairs (u_j, b_j) where the b_j's are in descending ordering.

Our next step is to obtain the OWA weights associated with this importance weighted aggregation. As discussed in Yager (1997) we obtain these weights as

$$w_j = Q\left(\frac{S_J}{T}\right) - Q\left(\frac{S_{j-1}}{T}\right)$$

where $S_j = \sum_k^j u_k$ and $T = \sum_{k=1}^n u_k$, the total sum of importances. Having obtained the weights we can now calculate the valuation $D(x)$ associated with x as $D(x) - \sum_{j=1}^n w_j b_j$. We emphasize that the weights used in this aggregation will generally be different for each

x. This is due to the fact that the ordering of the A_i's will be different and in turn lead to different u_j's and hence different w_j.

It can be shown in the special case where $Q(r) = r$, the unitor quantifier, this approach leads to the ordinary weighted average $D(x) = \frac{1}{T} \sum_{i=1}^{n} t_i A_i(x)$.

The following example illustrates the application of the above method

Example: We shall assume four agents A_1, A_2, A_3, A_4. The importances associated with these agents are $t_1 = 0.1$, $t_2 = 0.6$, $t_3 = 0.5$, $t_4 = 0.9$. We assume the support to alternate x given by each of the agents is: $A_1(x) = 0.7$, $A_2(x) = 1$, $A_3(x) = 0.5$ and $A_4(x) = 0.6$. We further assume that the quantifier guiding this mediation is **most** which is defined by $Q(r) = r^2$. In this case ordering the agent support for x give us

	b_j	u_j
A_2	1	0.6
A_1	0.7	0.1
A_4	0.6	0.9
A_3	0.5	0.5

We note that $T = \sum_{i=1}^{4} u_j = 3$. Using this we calculate the weights associated with x:

$$w_1 = Q\left(\frac{0.6}{3}\right) - Q\left(\frac{0}{3}\right) = (0.2)^2 - 0 = .04 \quad w_2 = Q\left(\frac{1.6}{3}\right) - Q\left(\frac{0.6}{3}\right) = .28 - .04 = .24$$
$$w_3 = Q\left(\frac{2.5}{3}\right) - Q\left(\frac{1.6}{3}\right) = .69 - .28 = .41 \quad w_4 = Q\left(\frac{3}{3}\right) - Q\left(\frac{2.5}{3}\right) = 1 - .69 = .31$$

In this case $D(x) = \sum_{j=1}^{4} w_j b_j = (.04)(1) + (.24)(.7) + (.41)(.6) + (.31)(.5) = .609$.

We now consider some of the properties associated with the introduction of agent importances in the mediation process. We first investigate how the OWA weights associated with the arguments change as we change the importance weights. In the following, without loss of generality, we assume the original agent indexing is consistent with the ordering.

Observation: If the importance weight of an agent increases, while the importances of the other agents remains the same, then its new OWA weights in the aggregation is at least as big as its original OWA weight.

Proof: Assume the original importance of the agents are u_j and the new importance weights are $\tilde{u}_j$, where $\tilde{u}_j = u_j$ for $j \neq q$ and $\tilde{u}_q = u_q + \Delta$ with $\Delta > 0$. Let $T = \sum_{j=1}^{n} uj$ and $S_i = \sum_{j=1}^{i} u_j$. In this situation the original OWA weight for agent q is $w_q = Q\left(\frac{S_q}{T}\right) - Q\left(\frac{S_{q-1}}{T}\right)$ and its new OWA weight is $\tilde{w}_q = Q\left(\frac{\tilde{S}_q}{\tilde{T}}\right) - Q\left(\frac{\tilde{S}_{q-1}}{\tilde{T}}\right)$ where $\tilde{T} = T + \Delta$, $\tilde{S}_{q-1} = S_{q-1}$ and $\tilde{S}_q = S_q + \Delta$. In this case

$$\tilde{w}_q = Q\left(\frac{S_q + \Delta}{T + \Delta}\right) - Q\left(\frac{S_{q-1}}{T + \Delta}\right)$$

We see that $\frac{S_{q-1}}{T} > \frac{S_{q-1}}{T+\Delta}$. Furthermore $\frac{S_q + \Delta}{T+\Delta} - \frac{S_k}{T} = \frac{T S_k + T \Delta - T S_K - \Delta S_k}{T(T+\Delta)} = \frac{\Delta(T - S_k)}{T(T+\Delta)} > 0$ and thus $\frac{S_q+\Delta}{T+\Delta} \geq \frac{S_k}{T}$. From the monotonicity of Q we see

$$\tilde{w}_q - w_q = Q\left(\frac{S_q + \Delta}{T + \Delta}\right) - Q\left(\frac{S_q - 1}{T + \Delta}\right) - \left(Q\left(\frac{S_q}{T}\right) - Q\left(\frac{S_q - 1}{T}\right)\right)$$

$$\tilde{w}_q - w_q = \left(Q\left(\frac{S_q + \Delta}{T + \Delta} \right) - Q\left(\frac{S_q}{T} \right) \right) + \left(Q\left(\frac{S_q - 1}{T} \right) - Q\left(\frac{S_q - 1}{T + \Delta} \right) \right) \geq 0 \qquad \square$$

Other than this situation the effect of a change in importances is strongly dependent upon Q, the type of mediation imperative being used.

Looking at the linear mediation imperative, $Q(x) = x$, allows us to obtain a deeper understanding of the effect of changes in importances. Assume each agent has an importance t_i and let $T = \sum_{i=1}^n t_i$. In the case of the linear mediation imperative we get $w_i = \frac{t_i}{T}$. Assume now that each of the agents has a Δ_i change in its importance. Here the new importance of each agent is $t_i + \Delta_i$. We shall let $\Delta = \sum_{i=1}^n \Delta_i$. In this case the new weight associated with the ith agent is $\tilde{w}_i = \frac{t_i + \Delta_i}{T + \Delta}$. Consider the ratio $\frac{\tilde{w}_i}{w_i} = \frac{\frac{t_i + \Delta_i}{T + \Delta}}{\frac{t_i}{T}} = \frac{t_i + \Delta_i}{T + \Delta} \frac{T}{t_i} = \frac{1 + \frac{\Delta_i}{t_i}}{1 + \frac{\Delta}{T}}$. We see that those agents in which $\frac{\Delta_i}{t_i} > \frac{\Delta}{T}$ will have an increase in weight. Those for which $\frac{\Delta_i}{t_i} < \frac{\Delta}{T}$ will have a loss in weight and those for which $\frac{\Delta_i}{t_i} = \frac{\Delta}{T}$ will remain the same.

It is interesting to consider the situation in the case in which we use the mediation function shown in Figure 6 to guide the aggregation. For simplicity we shall consider the situation in which our argument is the collection $[(b_1, \mu_1), \ldots, [b_n, \mu_n]$ where the b_i are in descending order, and the μ_i are normalized, $\sum_i \mu_i = 1$. In this case $w_j = Q(\mu_j) - Q(\mu_j - 1)$. For simplicity we shall also assume the $\sum_{j=1}^k \mu_j = \beta$, the sum of first k normalized importances exactly equals β. Here

$$w_j = 0 \text{ for } j = 1 \text{ to } k \quad \text{and} \quad w_j = \frac{\mu_j}{1 - \beta} \text{ for } j = k + 1 \text{ to } n$$

Thus the first k arguments have zero weight. The remaining weights are simply the agents importance weight normalized by $1 - \beta$. Thus for this mediation imperative we have

$$D(x) = \frac{1}{1 - \beta} \sum_{j=k+1}^n b_j u_j$$

8. Associating Importances with Agents

We now consider the process for assigning the importance to each of the agents in the mediation process. As we indicated our purpose in including importances is to reward those agents who are most open to solutions. With this in mind, one way to accomplish this is to associate with each agent an importance value equal to the sum of the scores in its preference function.

Consider the situation in which $X = \{x_1, \ldots, x_n\}$. Assume the kth agent has preference A_k, here $A_k(x_i)$ is the support he allocates to alternative x_i. We let $I_k = \sum_{i=1}^n A_k(x_i)$ be the importance weight associated with the kth agent. Using this we see the more open an agent is to solutions the more important the agent is in the mediation process. An agents importance charges at each stage in the negotiation as it depends on the preference function he supplies in that round. The following example illustrates this approach to rewarding agent openness.

Example: Assume two agents and four alternative $x = \{x_1, x_2, x_3, x_4\}$. Assume our mediation is based on the quantifier is $Q(y) = y$. Let $A_1 = \{\frac{1}{x_1}, \frac{0.5}{x_2}, \frac{0}{x_3}, \frac{0}{x_4}\}$ and $A_2 = \{\frac{1}{x_1}, \frac{0.3}{x_2}, \frac{0.7}{x_3}, \frac{1}{x_4}\}$. For agent 1 $I_1 = 1.5$ and for agent 2 $I_2 = 2$. The normalized importances are $\mu_1 = \frac{1.5}{3.5} = 0.43$ and $\mu_2 = \frac{2}{3.5} = 0.57$. We now calculate $D(x_j) = \frac{43}{.57} A_1(x_j) + .57 A_2(x_j)$. Here

	$\mathbf{D(x_j)}$
x_1	.0.43
x_2	0.385
x_3	0.4
x_4	.0.57 $\Leftarrow$

Thus the agent with most openness gets his preferred solution

We note that other formulations for calculating the importances can be considered. Let us look at some of these. One more generalized form is to define $I_k = \sum_{i=1}^{n} g(A_k(x_i))$ where g is monotonic. A particular example is where g is defined such that:

$$g(y) = 0 \quad \text{if } 0 \leq y < \alpha$$
$$g(y) = 1 \quad \text{if } \alpha < y \leq 1$$

Thus here if an agent supports an alternative with value greater than α than gets full credit for supporting it. Another formulation is to have $I_k = \sum_{i=1}^{n}(g(A_k(x_i))^b$ here we want be $b \in (0, 1]$.

The general idea we are proposing here is the rewarding of agents for being accommodating to different solutions in the negotiation process by correlating their importance with the quantity of solutions they are supporting.

9. Stopping Rule

An important part of the negotiation process is the stopping rule. This is the rule that determines when we stop the rounds of mediation, it decides the final group preference function. The form of the stopping plays an important part in the determination of the strategy that an agent uses in the negotiation process. Here we shall consider some examples of stopping rules.

One basic form of stopping rule is a fixed round of mediation. Here we initially select some integer value m and we stop after m rounds of mediation. Thus the group preference function obtained during the mth round of mediation is used to determine the group's choice of alternative.

Another class of stopping rules are those based on the idea that we should stop when some agreement has been reached by the participating agents. One type of agreement is with respect to the group's preferred alternative solution. One possible method for determining this is the following. Let $D: X \rightarrow [0,1]$ be the group preference function after the mediation

step. Let ρ be a permutation of the alternatives so that $x_{\rho(j)}$ is the alternative with the jth largest score in D. In this case $D(x_{\rho(j)})$ is the score of the alternative with the jth largest support. We now let $\Delta = x_{\rho(1)} - x_{\rho(2)}$, the difference between the two highest supports. Under this stopping rule we "a priori" select some value α and stop the rounds of mediation once we obtain $\Delta \geq \alpha$. In this case, $\Delta \geq \alpha$, we essentially have found some alternative that the group has agreed in the most preferences. One problem here is that this condition, $\Delta \geq \alpha$, may never be satisfied. Thus using this type of stopping rule may require some back-up rule, such as stop after m rounds. It is also possible to make α dependent upon the number of rounds k. In the case if $\alpha(k)$ is the threshold we use after k rounds then we require that $\alpha(k_1) \leq \alpha(k_2)$ if $k_1 > k_2$.

A second type of agreement is where the agents have concurred on a group preference function. One method of determining this is as follows. Let D_{k-1} and D_k be the group functions obtained after the $k-1$th and kth iteration. Let CLOSE $(D_k, D_{k-1}) \in [0, 1]$ indicate the degree of closeness between these two preference functions. If we a priori select some value δ then we say that the group has agreed on a preference a function if CLOSE$(D_k, D_k - 1) \geq \delta$. Thus we stop if the condition has been attained. This approach raises the issues of measuring the distance between these group preference functions. However, since these group preference functions can be viewed as fuzzy subsets we can draw up the copious literature on calculating the distance between fuzzy sets. We briefly describe some methods to calculate CLOSE$(D_k, D_k - 1)$.

Early work on calculating the distance between fuzzy subsets was presented in Kaufman (1975). The fundamental unit in discussing the distance between two fuzzy subsets, D_k and D_{k-1}, is the elementary difference $\Delta_j = |D_k(x_j) - D_{k-1}(x_j)|$. We note $\Delta_j \in [0, 1]$. Using this we can introduce a Hamming like measure of the distance between the two sets $\text{Dist}_1(D_k, D_k - 1) = \frac{1}{n} \sum_{j=1}^{n} \Delta_j$. We note $\text{Dist}_1(D_k, D_{k-1}) \in [0, 1]$.

A more general measure of the distance between two fuzzy subsets can be obtained if we use the generalized mean. In this case $\text{Dist}_\alpha(D_k, D_{k-1}) = (\frac{1}{n} \sum_{j=1}^{n} \Delta_j^\alpha)^{1/\alpha}$ where $\alpha \geq 1$. For $\alpha = 1$ we get the Hamming formula. For $\alpha = 2$ we get $\text{Dist}_2(D_k, D_{k-1}) = \text{Max}_j [\sum_{j=1}^{n} \Delta_j^2]^{1/2}$ which is a Euclidean type measure. For $\alpha \to \infty$ we get $\text{Dist}(D_k, D_{k-1}) = \text{Max}_j[\Delta_j]$, it is the largest of elementary differences.

Using this measure of Dist we can define our desired measure of closeness as

$$\text{CLOSE}(D_k, D_k - 1) = 1 - \text{Dist}(D_k, D_{k-1})$$

In the preceding we have introduced a definition for a fuzzy relationship Dist and then expressed CLOSE as **Not** Dist. In the following we shall suggest some alternative, more intelligent ways of defining the relationship CLOSE. Again we shall introduce a relationship Dist and define CLOSE as **Not** Dist, however our definition of Dist will be different then in the preceding

Here we shall say that D_k and D_{k-1} are distant if there exists **at least** Q elements in X that have a membership grades in D_k and D_{k-1} that are far from each other. Individually we can express the degree of farness of an element's membership grade as $\Delta_j = |D_k(x_j) - D_{k-1}(x_j)|$. We shall define the quantity Q using a fuzzy subset Q over the space of non-negative integers N. Under this definition $Q(j)$ indicates the degree to

which j elements satisfy the concept of "at least Q". We note that Q must be monotonic, $Q(j) \geq Q(i)$ if $i > j$. Before proceeding we introduce a notational convention. We let index(k) be the index of the kth largest Δ_j, thus $\Delta_{\text{index(k)}}$ is the value of the kth largest distance between the membership grades of the two fuzzy subsets. Using this we express the relationship Dist as

$$\text{Dist}(D_k, D_{k-1}) = \text{Max}\, j[Q(j)\Delta_{\text{index}(j)}]$$

In the preceding we used as the measure of farness between two membership grades, $D_k(x_j)$ and $D_{k-1}(xj)$, the value Δ_j. A further degree of sophistication can be added if we introduce a fuzzy subset FAR on the unit interval such that $\text{FAR}(0) = 0$, $\text{FAR}(1) = 1$ and $\text{FAR}(a) \geq \text{FAR}(b)$ if $a > b$. Using this we can modify our definition of Dist to be Dist $(D_k, D_{k-1}) = \text{Max}_j[Q(j)\text{FAR}(\Delta_{\text{index}(j)})]$.

In introducing FAR we are allowing for a refinement of the concept of two membership grades being far apart. For example we can define FAR such that:

$$\text{FAR}(\Delta) = 0 \text{ if } \Delta \leq a \quad \text{FAR}(\Delta) = \frac{\Delta - a}{b - a} \text{ if } a \leq \Delta \leq b \quad \text{FAR}(\Delta) = 1 \text{ if } \Delta \geq b$$

Here we allow some range, less than a, where we consider the difference insignificant, while for $\Delta \geq b$ we assume farness is complete. We can also make the transition non-linear by defining

$$\text{FAR}(\Delta) = 0 \text{ if } \Delta < a \quad \text{FAR}(\Delta) = \left(\frac{\Delta - a}{b - a}\right)^p \text{ if } a \leq \Delta \leq b \quad \text{FAR}(\Delta) = 1 \text{ if } \Delta \geq b$$

where $p \geq 0$. We see that if $p \to 0$ this essentially reduces to a binary definition with boundary at $\Delta = a$. If $p \to \infty$ we also get a binary definition with boundary at $\Delta = b$.

One problem with using this type of measure of closeness as our stopping rule is that we may never obtain two adjacent group preference functions that are close enough to each other to stop the process. One way to deal with this problem is to make the requirements for closeness dependant upon the number of iterations the negotiation process has gone through. Let us take a brief look at how to formulate this. In the following we shall let r indicate the number of rounds of mediation. Using our definition of closeness, $\text{CLOSE}(D_k m D_{k-1}) = 1 - \text{Distant}(D_k, D_{k-1})$ where $\text{Distant}(D_k, D_{k-1}) = \text{Max}_j[Q(j \cdot \text{FAR}(\Delta_{\text{index}(j)})]$ we stop when CLOSED, $(D_k, D_{k-1}) \geq \delta$ or when Dist $(D_k, D_{k-1}) \leq 1 - \delta$.

Our goal in considering the number of rounds in the negotiation process is to make things appear closer as we increase r. This means we want to reduce the distance between fuzzy subsets as the value of r increases. Essentially if we let $\text{Dist}_r(D_r, D_{r-1}) = \text{Max}_j[Q_r(j) \cdot \text{FAR}(\Delta_{\text{index}(j)})]$ indicate the distance between D_k and D_{k-1} after the rth round we want

$$\text{Dist}\, r_2(A, B) \leq \text{Dist}_{r_1}(A, B) \text{ if } r_2 > r_1.$$
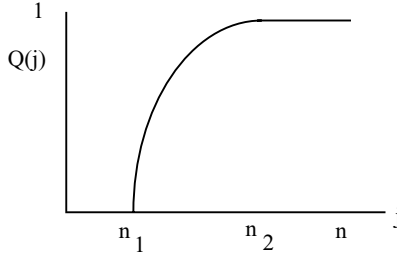
Figure 9. **Prototypical form for Q**

Essentially we want $Q_{r_2}(j) \leq Q_{r_1}(j)$ for $r_2 > r_1$. Thus we want Q to be a decreasing function of r. In Figure 9 we show the prototypical form for Q_r. One approach to obtaining our objective is to define Q_r as follows

$$Q_r(j) = 0 \quad \text{if } j \leq n_1(r) \qquad Q_{r(j)} = \frac{j - n_1(r)}{n_2(r) - b} \quad \text{if } n_1(r) \leq j \leq n_2(r)$$

$$Q_{1(j)} = 1 \text{ if } j \geq n_2(r)$$

We then let $n_1(r)$ and $n_2(r)$ be increasing functions of r.

Here we describe another method for terminating the rounds of mediation. This can be seen to be in the spirit of the game of "musical chairs" Assume that at the end of the kth round of mediation D_k is the group preference over the set X. Assume that alternative x_q is such that $D_k(x_q) = \text{Min}_{x \in X}[D(x)]$, it is the alternative least preferred by the group. In this method we eliminate x_q from the set of possible alternatives. So in this situation the set of possible alternatives changes in each round, it gets reduced by one. We note that this stopping rule results in a fixed number of mediation rounds. If we start with n alternatives we need $n - 1$ mediation rounds to get down to one element. One issue that we must address is if two or more elements are tied with the lowest score. Here we suggest using a random mechanism to select one among these to be eliminated.

10. Information Availability

We must emphasize here the distinction between a participating agent's valuation function V_k and the preference function A_k he provides to the mediation at a given step in the process. The preference function is a mapping $A_k : X \rightarrow [0, 1]$, that associates with each alternative a value in the unit interval corresponding to the support he is giving to the alternative. It is dynamic and can change at each round in the negotiation process. The form of A_k is guided by the strategy the agent is using in the negotiation process. It is informed by the agent's own valuation function as well as any information the agent may have about the other agent's preferences. We note that an agent's valuation function, V_k, is assumed, although this is not necessary, to be static and unchanging. The valuation function reflects his perceived worth

of an alternative. It is noted that the form of this can be very precise or extremely imprecise. An agent may have an exact quantitative value for each $x \in X$. It may only be a linear ordering. It can be a very imprecise unquantified image possessed by the agent. All that is necessary however is that the agent provides a precise preference function. In addition to having information about his own value function each agent should be provided with some information about the negotiation process. It appears that at the very least each agent should be provided with the group preference function that results after each mediation round. Another possibility is that each agent is given the preference functions provided by the other agents at each round of mediation. This is clearly more informative. Part of the formulation of the negotiation protocol is the determination of what type of information is provided. Another issue is whether individual agents can exchange information between themselves.

We noted that the strategy an agent uses to provide his preference function is informed by what he knows about the other agents preferences. The source of this is the data supplied about the group preferences and the other agents preference functions in preceding rounds. However, the task of extracting knowledge from this data in a form that is useful for the determination its subsequent preference functions is generally quite difficult and not within our current interest. But this is an important problem.

11. Selection Process

In fourth step of our negotiation algorithm we select the alternative based upon the final group preference function. Thus here our point of departure is a mapping $D: X \rightarrow [0, 1]$ where $D(x)$ is the groups support for alternative x. The most obvious procedure here is to select alternative x^* such that $D(x^*) = \text{Max}_{x \in X}[D(x)]$, that is we select the alternative with the most support. We shall refer to this as the standard selection process. We note that with this standard selection process if two alternatives are tied we must use a random selection to choose between them, a coin toss. In many ways this standard selection process is extremely brittle. Consider the situation in which x_1 has the maximum score $D(x_i) = .83$ while $D(x_2) = .825$ here we would select x_1. However a small change in $D(x_2)$ could make these tied or even cause of selection of x_2 over x_1. Given the very subjective and imprecise nature of the manner in which each individual agent determines its preference function this type of brittle selection process may not be always advisable. In the following we shall suggest some alternate methods for selecting the winning alternative from the final group preference function.

One method is based upon the use of a parameter $\lambda > 0$, although normally we use $\lambda \geq 1$. In this method we proceed as follows. We first associate with each $x \in X$ a probability

$$P(x_i) = \frac{D(x_i)^\lambda}{\sum_j D(x_j)^\lambda}$$

We then randomly select our selected alternative using a biased random experiment with these probabilities. We note that if $\lambda \rightarrow \infty$ then $P(x^*) = 1$ if $D(x^*) = \text{Max}_{x \in X} D(x)$ and

$D(x_j) = 0$ if $x_j \neq x^*$. This is essentially what we called the standard selection process. Parenthetically we note in this case if q alternatives are tied where the biggest value then they each get probability $\frac{1}{q}$ while all the others are assigned probability zero. If $\lambda = 1$ then $P(x_i) = \frac{D(x_i)}{\sum_j D(x_j)}$ it is simply the normalization of the scores. We note that if $\lambda > 0$ then any x_j with $D(x_j) = 0$ has $P(x_j = 0)$ and has no chance of being selected. In this approach we can see that λ is an indication of the *significance* we give to the final group preference function in the selection process. If $\lambda \to \infty$ we select the alternative that it says is best.

In this approach using D we get the probability $P(x_j)$, the probability of selecting x_j. If agent i has valuation function V_i, $V_i(x_j)$ being the value of alternative x_j to the ith agent, then for this agent the expected value associated with group preference function D is $\overline{V}_i = \sum_{j=1}^{n} P(x_j)V_i(x_j)$. In the special case when $\lambda \to \infty$ then with x^* being the maximally supported alternative we get $\overline{V}_i = V(x^*)$, the value of x^* to him.

12. Agent Strategy

Once having been informed of the protocol of the negotiation process each agent must determine a strategy to use in providing its preference function at each stage in the negotiation process. At a meta level the basis of this strategy is the agent's objective to trying to get the best payoff. If the agent can express his individual payoff function as numeric value then the payoff to the agent is

$$\widetilde{v}_i = \sum_{j=1}^{n} \widetilde{p}_j v_i(x_j)$$

where $v_i(x_j)$ is the value of alternative x_j to agent i and $\widetilde{p}_j$ is the probability of selecting alternative x_j at the end of the rounds of negotiation. The actual determination of $\widetilde{p}_j$ is a complex task as it depends on the final group preference function. Thus not only is the value of the $\widetilde{p}_j$'s dependent upon the protocol of the negotiation process but it also depends upon the strategies used by each of the participating agents, more specifically the preference functions provided by each agent at each stage. In addition any external information available about the other agent's preferences may effect the process.

It is the role of what we shall call an agents ***negotiation consultant*** to help determine the best strategy for this agent. In some cases, where the structure of the negotiation process is simple, the determination of a precise optimal strategy may be possible. More often the problem of determining an optimal strategy is very difficult with much uncertainty as it requires knowledge and judgment about what others will do. It is a game theoretic problem with incomplete information. More realistically, the best we can hope for is some approximate and imprecise rules to help guide an agent in the process. We feel that the construction of these type of rules guiding an agent will require the use

of the type of tools available in Zadeh's (1979) fuzzy set based theory of approximate reasoning.

Here we shall look at a very basic simple situation and see if we can develop some strategies for our agent. We assume our agent has a well defined preference function, $V_1(x_j)$ is the payoff for x_j. We shall assume a two agent negotiation. We shall assume a linear mediation process. Thus if A_1 and A_2 are two agent preference function the group preference D is

$$D(x_j) = \frac{I_1}{I_1 + I_2} A_1(x_j) + \frac{I_2}{I_1 + I_2} A_2(x_j)$$

where $I_i = \sum_{j=1}^{n} A_i(x_j)$. We shall assume a one round negotiation process, thus our stopping rule is $m = 1$. Finally we shall assume that the alternative selection is determined in a probabilistic manner, based on the value in the final group preference function. Thus if $\widetilde{D}$ is the final group support the probability of selecting x_j is $P_j = \frac{\widetilde{D}(x_j)}{\sum_{i=1}^{n} \widetilde{D}(x_i)}$. Since we are allowing only one round of negotiation then if A_1 and A_2 are preference functions provided by agents we have $\widetilde{D}(x_j) = \frac{I_1}{I_1 + I_2} A_1(x_j) + \frac{I_2}{I_1 + I_2} A_2(x_j)$ and hence $\widetilde{P}_j = \frac{I_1 A_1(x_j) + I_2 A_2(x_j)}{\sum_{i=1}^{n} I_1 A_1(x_i) + I_2 A_2(x_i)}$. Using this we the expected payoff to agent one as

$$\overline{V}_1 = \frac{\sum_{j=1}^{n} (I_1 A_1(x_j) + I_2 A_2(x_j) V_1(x_j)}{\sum_{i=1}^{n} I_1 A_1(x_i) + I_2 A_2(x_i)}$$

where $I_i = \sum_{j=1}^{n} A_i(x_j)$.

As the negotiation consultant to agent 1 our task is to tell him what is the structure of A_1, the values to be used for the $A_1(x_j)$. The solution to this task involves finding $A_1(x_j)$ for $j = 1$ to n such that $\frac{\partial \overline{V}_1}{\partial A_1(x_j)} = 0$ for $j = 1$ to n. An operational solution to this eventually requires that the consultant knows or is able to estimate $A_2(x_j)$, the preference function that will be supplied by the other agent.

13. Conclusion

We considered the problem of multi-agent negotiation and provided an framework in which each of the participating agents provides a preference function over the set of alternatives. This framework required a mediation step in which the individual agent preference functions are aggregated to obtain a group preference function. It also involved a stopping rule which decided whether an acceptable group preference function has been obtained or whether the agents must participate in another round of mediation. It also contained a selection procedure for choosing a alternative based on the final group preference function. We described various implementations for these different steps. Considerable interest was focused on the implementation of the mediation rule where we allowed for a linguistic description of the preference aggregation rule. A particularly notable feature of our approach was the inclusion in the mediation step of a

mechanism rewarding the agents for being open to alternatives other then simply their most preferred.

Notes

1. For our primary focus, the formulation of a framework for negotiations, the exact nature of this mapping need not be known. All that we want to assume is that each agent has some means for formulating a preference function over the set of alternatives. We note that when considering the task of developing agent strategies this becomes of greater importance…
2. We note that more precisely we should use the notation A_{ik} indicating the ith agent's preference function on the kth round of mediation and use D_k. However, we shall suppress the index indicating the round.

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Prioritized Aggregation Operators

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ABSTRACT

We consider criteria aggregation problems where there exists a prioritization relationship over the criteria. We suggest that prioritization between criteria can be modeled by making the weights associated with a criteria dependent upon the satisfaction of the higher priority criteria. We consider a number of aggregation operators in which there exists a prioritization relationship between the arguments. We first introduce a prioritized averaging operator. We next introduce a prioritized "anding" and then a prioritized "oring" operator.

1. Introduction

Many applications involve the selection or ordering of a group of alternatives based upon their satisfaction to a collection of criteria. Typical examples of this are information retrieval , multi-criteria decision making and database retrieval. Search engines such as Google require the solution of this type of problem.

In these problems we have a collection of criteria $C = \{C_1, ..., C_n\}$ and a set of alternatives $X = \{x_1, ..., x_m\}$. We further have a measure of the satisfaction of criteria C_i by each alternative, $C_i(x) \in [0, 1]$. One commonly used approach is to calculate for each alternative x a score $C(x)$ as an aggregation of the $C_i(x)$,

$$C(x) = F(C_1(x), ..., C_n(x))$$

and then order the alternatives using these scores. The form for F depends upon the users desired imperative for performing this aggregation. A commonly used form for F is a weighted average of the $C_i(x)$. In particular we calculate

$$C(x) = \sum_{i = 1}^{n} w_i C_i(x)$$

where the weights satisfy $w_i \in [0, 1]$ and sum to one.

It is easy to see that this type of aggregation is monotonic, $C(x)$ does not decrease if any of the $C_i(x)$ increases. It is also bounded

$$Min_i[C_i(x)] \leq C(x) \leq Max[C_i(x)).$$

It is also idempotent, if all $C_i(x) = a$ then $C(x) = a$. Because of these properties this is an averaging operator.

Central to this type of aggregation is the ability to trade off between criteria. In particular $\frac{w_k}{w_i}$ is the relation between criteria C_i and C_k. In this type of aggregation we can compensate for a decrease of Δ in satisfaction to criteria C_i by gain $\frac{w_k}{w_i}\Delta$ in satisfaction to criteria C_k.

In many real applications we do not want to allow this kind of compensation between criteria. Consider the situation in which we are selecting a bicycle for our child based upon the criteria of safety and cost. In this situation we may not allow a benefit with respect to cost to compensate for a loss in safety. Here we have a kind of *prioritization* of the criteria. Safety has a higher priority. Consider a problem of document retrieval in which we are looking for documents about the American revolution and prefer if they are from an academic website and written after 2003. In this case the condition of it being about the American revolution has a priority, if it is not about this topic we not interested. In organizational decision making criteria desired by superiors generally have a higher priority then those of their subordinates.

In this work we shall suggest an averaging type aggregation operator that allows for the inclusion of priority between the criteria. Central to our approach will be the modeling of priority by using a kind of importance weight in which the importance of a lower priority criteria will be based on its satisfaction to the higher priority criteria [1]. As we shall see this result in a situation in which importance weights will not be the same across the alternatives.

2. A Prioritized Averaging Operator

In the following we assume that we have a collection of criteria partitioned into q distinct categories, $H_1, H_2, ..., H_q$ such that $H_i = \{C_{i1}, C_{i2}, ..., C_{in_i})$, Here C_{ij} are the criteria in category H_i. We assume a prioritization between these categories

$$H_1 > H_2, ... > H_q.$$

The criteria is class H_i have a higher priority than those in H_k if $i < k$. The total number of criteria is $C = \bigcup\limits_{i=1}^{q} H_i$. We assume $n = \sum\limits_{i=1}^{q} n_i$ the total number of criteria.

In figure #1 we show the positioning of the criteria

$$C_{11}, C_{12}, ..., C_{1n_1}$$

$$C_{21}, C_{22}, ..., C_{2n_2}$$
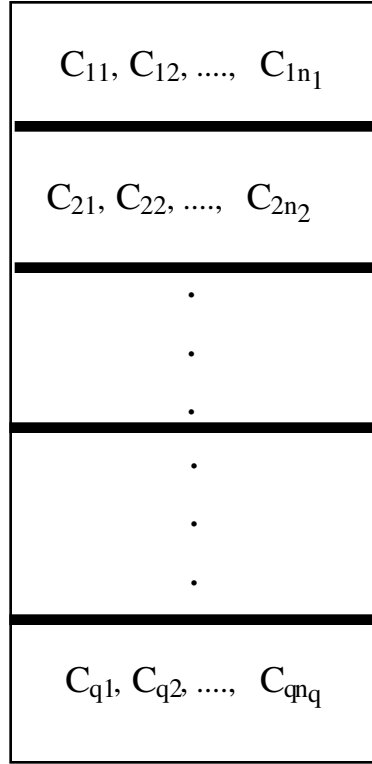
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$$C_{q1}, C_{q2}, ..., C_{qn_q}$$

Figure #1. Prioritization of Criteria

We assume that for any alternative $x \in X$ we have for each criteria C_{ij}, a value $C_{ij}(x) \in [0, 1]$ indicating its satisfaction to criteria C_{ij}.

In the following we introduce an aggregation operator which we refer to as the **Pri**oritized **Ave**raging (**PRI-AVE**) aggregation operator which allows us to calculate $C(x)$ for any alternative.

The form of our aggregation operator is

$$C(x) = \sum\limits_{i, j} w_{ij} \, C_{ij}(x)$$

Here the weights will also be a function of x and will be used to reflect the priority relationship. In order to obtain the weights for a given alternative x we proceed as follows.

For each priority category H_i we calculate

$$S_i = Min_j[C_{ij}(x)]$$

Here S_i is the value of the least satisfied criteria in category H_i under alternative x. Using this we will associate with each criteria C_{ij} a value u_{ij} called its pre-weight. In particular for those criteria in category H_1 we have $u_{1j} = 1$. For those criteria in category H_2 we have $u_{2j} = S_1$. For those criteria in category H_2 we have $u_{3j} = S_1S_2$. For those criteria in category H_4 we have $u_{4j} = S_1S_2S_3$. More generally u_{ij} is the product of the least satisfied criteria in all categories with higher priority than H_i.

We can more succinctly and more generally express $u_{ij} = T_i$ where

$$T_i = \prod_{k=1}^{i} S_{k-1}$$

with the understanding that $S_0 = 1$ by default. We note that we can also express T_i as

$$T_i = S_{i-1} T_{i-1}$$

This equation along with the fact that $T_1 = S_0 = 1$ gives a recursive definition at T_i.

We now see that for all $C_{ij} \in H_i$ we have $u_{ij} = T_i$. Using this we obtain for each C_{ij} a weight w_{ij} with respect to alternative x such that

$$w_{ij} = \frac{u_{ij}}{\sum\limits_{i=1}^{q} \sum\limits_{j=1}^{n_i} u_{ij}}$$

Since $u_{ij} = T_i$.this simplifies to $w_{ij} = \dfrac{T_i}{\sum\limits_{i=1}^{q} n_i T_i}$. Furthermore if we denote $T = \sum\limits_{i=1}^{q} n_i T_i$ this further simplifies to

$$w_{ij} = \frac{T_i}{T}.$$

It is easily seen that the $w_{ij} \in [0, 1]$ and sum to one..

Using these weights we then can get an aggregated score x under these prioritized criteria as

$$C(x) = \sum_{i,j} w_{ij} C_{ij}(x) = \frac{1}{T} \sum_{i,j} T_i C_{ij}(x)$$

Before studying at the properties of this aggregation technique we look at an example

Example: Consider the following prioritized collection of criteria:

$$H_1 = \{C_{11}, C_{12}\}$$

$H_2 = \{C_{21}\}$

$H_3 = \{C_{31}, C_{32}, C_{33}\}$

$H_4 = \{C_{41}, C_{42}\}$

Assume for alternative x we have

$C_{11}(x) = 0.7, C_{12} = (x) = 1$

$C_{21}(x) = 0.9$

$C_{31}(x) = 0.8, C_{23}(x) = 1, C_{33}(x) = 0.2$

$C_{41}(x) = 1, C_{42}(x) = 0.9$

We first calculate

$S_1 = \text{Min}[C_{11}(x), C_{12}(x)] = 0.7$

$S_2 = \text{Min}[C_{21}(x)] = -.9$

$S_3 = \text{Min}[C_{31}(x), C_{32}(x), C_{33}(x)] = -.2$

$S_4 = \text{Min}[C_{41}(x), C_{42}] = 0.9$

Using this we get

$T_1 = 1$

$T_2 = S_1 T_1 = 0.7$

$T_3 = S_2 T_2 = (0.9)\,(0.7) = 0.63$

$T_4 = S_3 T_3 = (0.2)(0.63) = 0.12$

From this we obtain

$u_{11} = u_{12} = T_1 = 1$

$u_{21} = T_2 = 0.7$

$u_{31} = u_{32} = u_{33} = T_3 = 0.63$

$u_{41} = U_{42} = T_4 = 0.12$

We obtain

$$T = \sum_{i,j} u_{ij} = \sum_{i=1}^{4} n_i T_i = 2T_1 + T_2 + 3T_3 + 2T_4$$

$T = (2)(1) + 0.7 + (3)(0.63) + (2)(0.12) = 4.83$

From this we get

6

$$w_{11} = w_{12} = \frac{T_1}{T} = \frac{1}{4.83} = 0.20$$

$$w_{21} = \frac{T_2}{T} = \frac{0.7}{4.83} = 0.145$$

$$w_{31} = w_{32} = w_{33} = \frac{T_3}{T} = \frac{0.63}{4.83} = 0.13$$

$$w_{41} = w_{42} = \frac{T_4}{T} = \frac{0.12}{4.83} = 0.025$$

We now calculate

$$C(x) = \sum_{ij} w_{ij} C_{ij}(x) = \frac{1}{4.83}[(C_{11}(x) + (C_{12}(x)) + 0.7(C_{21}(x)) + 0.63(C_{31}(x) + C_{32}(x)$$

$$+ C_{33}(x)) + 0.12(C_{41}(x) + C_{42}(x))]$$

$$C(x) = \frac{1}{4.83}(1.7 + (0.7)(0.9) + (0.63)(2) + (0.12)(1.9) = \frac{3.818}{4.83} = .79$$

We now investigate the properties of this aggregation method. First we see that this method is idempotent and bounded by the maximum and minimum of the arguments. For simplicity let us denote $a_{ij} = C_{ij}(x)$. Using this we have $C(x) = \sum_{i,j} w_{ij} a_{ij}$. First consider the case where all the a_{ij} are the same, $a_{ij} = d$. In this case since $\sum_{ij} w_{ij} = 1$ we get $C(x) = \sum_{i,j} w_{ij} d = d$ and hence the operation is idempotent.

Consider now boundedness. Assume $a = \text{Min}_{ij}[a_{ij}]$ and $b = \text{Max}_{ij}[a_j]$ then

$$C(x) = \sum_{i,j} w_{ij} a_{ij} \geq \sum_{i,j} w_{ij} a \geq a$$

and

$$C(x) = \sum_{i,j} w_{ij} a_{ij} \leq \sum_{i,j} w_{ij} b \geq b.$$

$$C(x) = \sum_{ij} w_{ij} a_{ij} \leq \sum_{ij} w_{ij} b \leq b$$

We now consider the issue of monotonicity. For simplicity we shall look at the situation in which there is only one criteria at each priority level. Here we denote the criteria C_j, $j = 1$ to q. We shall denote the satisfaction of each criteria to x as $a_j = C_j$. We note that in this case with one criteria at each level, $S_i = a_i$. Here then $T_1 = 1$, $T_2 = a_1$ and more generally $T_i = \prod_{k=1}^{i} S_{k-1}$. Using this we have

$$C(x) = \frac{\sum_{i=1}^{q} T_i a_i}{T}$$

Let us denote $C(x) = \dfrac{M}{T}$ where $M = \displaystyle\sum_{i=1}^{q} T_i\, a_i$ and $T = \displaystyle\sum_{i=1}^{q} T_i$. . For monotonicity to hold we have to show that $\dfrac{\partial C(x)}{\partial a_j} \geq 0$ for any j. This requires that

$$\frac{T\dfrac{\partial M}{\partial a_j} - M\dfrac{\partial T}{\partial a_j}}{(T)^2} \geq 0.$$

Hence we must show that the numerator is non-negative,

$$T\frac{\partial M}{\partial a_j} - M\frac{\partial T}{\partial a_j} \geq 0.$$

Before preceding we note that $\dfrac{\partial T_i}{\partial a_j} = 0$ for $i \leq j$ and $\dfrac{\partial T_i}{\partial a_j} = \dfrac{T_i}{a_j}$ for $i > j$. We also note that

$$M = \sum_{i=1}^{q} T_i\, a_i = \sum_{i=1}^{q} T_{i+1} \text{ since } T_i\, a_i = T_{i+1}.$$ However we shall find it more useful to express

$$M = \sum_{i=2}^{q+1} T_i.$$

We shall denote $A = \dfrac{\partial M}{\partial a_j} = \dfrac{1}{a_j}\displaystyle\sum_{i=j+1}^{q+1} T_i$. We shall also let $B = \dfrac{\partial T}{\partial a_j}$ hence since $T = \displaystyle\sum_{i=1}^{q} T_i$

we have $B = \dfrac{\partial T}{\partial a_j} = \dfrac{1}{a_j}\displaystyle\sum_{i=j+1}^{q} T_i$. From this we observe that $A \geq B$. In the following we shall find it

convenient to denote $E = \displaystyle\sum_{i=2}^{j} T_i$.

Consider now the term $T\dfrac{\partial M}{\partial a_j} - M\dfrac{\partial T}{\partial a_j} = AT - BM$. We now observe that

$$T = \sum_{i=1}^{q} T_i = \sum_{i=1}^{j} T_i + a_j\, B.$$

Since $T_1 = 1$ then $T = 1 + E + a_j\, B$. We further observe that

$$M = \sum_{i=2}^{q+1} T_i. = E + a_j\, A.$$

Using the relations we see that

$$AT - BM = A(1 + E + a_j\, B) - B(E + a_j\, A) = A + EA + a_j\, BA - BE - a_j\, BA$$

$$AT - BM = A + E(A - B)$$

Since $A \geq B$ it follows that $AT - BM \geq 0$

The proof of monotonicity in the case where we can have multiple criteria at each level, although slightly more complicated, follows in the same spirit

We now look at some further properties of the proposed aggregation method. We recall $H_i = \{C_{ij} \mid j = 1 \text{ to } n_i\}$ where the criteria in category H_i have priority over those in H_k if $i_i\, i < k$.

Again letting $a_{ij} = C_{ij}(x)$ we have $S_i = \text{Min}_j[a_{ij}]$ and $S_o = 1$ and $T_i = \prod_{k=1}^{i} S_{k-1}$. Here with $u_{ij} = T_i$

and $T = \sum_{i=1}^{q} n_i\, T_i$ we have $w_{ij} = \dfrac{u_{ij}}{T} = \dfrac{T_i}{T}$ we have

$$C(x) = \sum_{i=1}^{q} \left(\sum_{j=1}^{n_i} w_{ij}\, a_{ij} \right) = \frac{1}{T} \sum_{i=1}^{q} T_i \left(\sum_{j=1}^{n_i} a_{ij} \right)$$

Letting $A_i = \sum_{j=1}^{n_i} a_{ij}$ we have

$$C(x) = \frac{1}{T} \sum_{i=1}^{q} T_i\, A_i$$

We see that the weight associated with the elements in the ith category are $\dfrac{T_i}{T}$ where

$T_i = \prod_{k=1}^{i} S_{k-1}$. Thus the criteria in H_i contribute proportion to the product of the satisfaction of the higher order criteria. Thus poor satisfaction to any higher criteria reduces the ability for compensation by lower priority criteria.

We also observe that if there exists some category H_r such that $C_{rj}(x) = 0$ for some criteria in H_r. then $S_r = 0$ and $T_i = 0$ for $i > r$ and hence $C(x) = \dfrac{1}{T} \sum_{i=1}^{r} T_i\, A_i$.

Note: While in the preceding we assumed $C_{ij}(x) \in [0, 1]$ this is not necessarily required. If we let $F_{ij}\, R \to [0, 1]$ be some function from the real numbers into the unit intervals such that $F_{ij}(C_{ij}(x))$ is some measure of how satisfied we are with a score $C_{ij}(x)$ for criteria C_{ij} then we allow the values of $C_{ij}(x)$ be any number if we calculate

$$S_i = \text{Min}_j[F_{ij}(C_{ij}(x)]$$

Here we just transfer the $C_{ij}(x)$ into numbers in the unit interval for calculating S_i.

3. Alternative Determination of Weights

In the preceding we introduced a prioritized multi-criteria aggregation method in which our criteria where partitioned into q categories, $H_i = \{ C_{ij}/j = 1 \text{ to } n_i \}$ where category H_i had priority

over H_k if $i < k$. For a given alternative x we shall find it convenient in the following to denote $C_{ij}(x)$ $= a_{ij}$. Using this notation then we defined

$$S_o = 0$$

$$S_i = Min_j[a_{ij}] \text{ for } i = 1 \text{ to } q$$

$$T_i = \prod_{k=1}^{i} S_{k-1} \text{ for } 1 = \text{ to } q$$

$$T = \sum_{i=1}^{q} n_i T_i$$

With $w_{ij} = \dfrac{T_i}{T}$ we obtained as our aggregated value

$$C(x) = \sum_{i=1}^{q} \sum_{j=1}^{n_i} w_{ij} a_{ij} = \frac{1}{T} \sum_{i=1}^{q} \sum_{j=1}^{n_i} T_i a_{ij}$$

Letting $A_i = \sum_{j=1}^{n_i} a_{ij}$ we can express this as $C(x) = \dfrac{1}{T} \sum_{i=1}^{n} T_i A_i$.

In the preceding we assumed that the satisfaction to the priority class $H_i = \{C_{i1}, ..., C_{in_i}\}$ under alternative x was determined by the least satisfied criteria in H_i, $S_i = Min_j[C_{ij}(x)]$. Here we shall suggest some alternative methods for calculating S_i.

One method we shall consider will be based on the OWA aggregation operator [2, 3]. Here we associate with each priority class H_i a vector $\mathbf{V_i}$ of dimension n_i called the OWA weighting vector. The components V_{ik} of $\mathbf{V_i}$ are such that $V_{ik} \in [0, 1]$ and $\sum_{k=1}^{n_i} V_{ik} = 1$. Additionally we let $ind_i(k)$ be an index of function so that $b_{ik}(x) = C_{ind_i(k)}(x)$ is the kth largest of $C_{ij}(x)$. Using this we now calculate

$$S_i = \sum_{k=1}^{n_i} V_{ik} \, b_{ik}(x)$$

We see that if $V_{in_i} = 1$ and $V_{ik} = 0$ for $k \neq n_i$ then we get $S_i = Min_j[C_{ij}(x)]$, the original method. An important special case is where $V_{ik} = 1/n_i$ for all k. In this case $S_i = \dfrac{1}{n_i} \sum_{j=1}^{n_i} C_{ij}(x)$. Here we take as S_i the average of the satisfactions of the criteria in category H_i. Another special case is when $V_{il} = 1$ and $V_{ik} = 0$ for $k \neq 1$. In this case $S_i = Max_j[C_{ij}(x)]$. Here we take S_i as the score of the most satisfied criteria in category H_i. Many other weight vectors are possible for example if $V_{iq} = 1$

for some q S_i simply becomes the qth largest of the $C_{ij}(x)$.

In this framework we can associate with each weighing vector $\mathbf{V_i}$ a measure called its attitudinal character denoted, A-C($\mathbf{V_i}$) [4]. We define this as

$$\text{A-C}(\mathbf{V_i}) = \frac{1}{n_i - 1} \sum_{k=1}^{n_i} V_{ik} (n_i - k)$$

It can easily be shown [2] that for the case where $V_{in_i} = 1$ we get A-C($\mathbf{V_i}$) = 0. For the case where A-C($\mathbf{V_i}$) = $\frac{1}{n_i - 1}$ then A-C($\mathbf{V_i}$) = 0.5 and for the case where $V_{i1} = 1$ we have A-C($\mathbf{V_i}$) = 1.

If we denote A-C(V_c) = α_i then we see in figure #2 the relationship between the value of α_i and the form for the calculation of S_i. Here then α_ι can be seen as a measure of the *tolerance* in determining the satisfaction of the category. While it is not necessary, it would be seen that the default situation is to assume α_ι is the same for all H_i.

α_i 0   0.5   1

$S_i = \text{Min}_j [C_{ij}(x)]$ $S_i = \text{Ave}_j [C_{ij}(x)]$ $S_i = \text{Max}_j [C_{ij}(x)]$
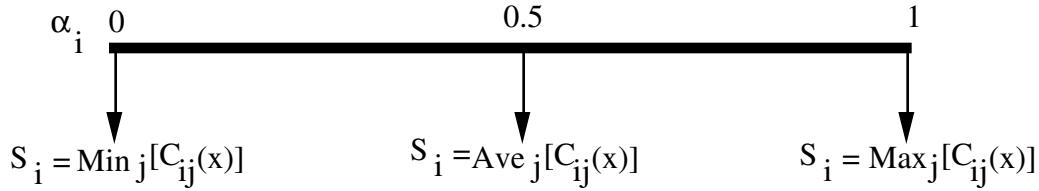
Figure #2. Relationship between α_i and the form of S_i.

Many of the techniques available for calculating the OWA weights [5] can be tailored for this particular application. A particularly interesting possibility is to use a variation of the method originally suggested by O'Hagan [6- 8]. In this case we would supply a desired level of tolerance α_i and solve the following mathematical programming problem for the V_{ik}

$$\text{Min} \sum_{K=1}^{n_i} (V_{iK})^2$$

$$\text{Such that: } \frac{1}{n_i - 1} \sum_{k=1}^{n_i} V_{ik} (n_i - k) = \alpha_i$$

$$\sum_{k=1}^{n_i} V_{ik} = 1$$

$$V_{ik} \geq 0$$

We provide an of the preceding variation using the earlier example

Example: $H_1 = \{C_{11}, C_{12}\}$, $H_2 = \{C_{21}\}$, $H_3 = \{C_{31}, C_{32}, C_{33}\}$. $H_4 = \{C_{41}, C_{42}\}$

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For alternative x we have

$$C_{11}(x) = 0.7, C_{12}(x) = 1$$

$$C_{21}(x) = 0.9$$

$$C_{31}(x) = 0.8, C_{32}(x) = 1, C_{33}(x) = 0.2$$

$$C_{41}(x) = 1 \qquad C_{42}(x) = 0.9$$

Consider the case where $S_i = \text{Max}_j[C_{ij}(x)]$. Here then

$$S_1 = 1$$

$$S_2 = 0.9$$

$$S_3 = 1$$

$$S_4 = 1$$

From this we get :

$$T_1 = 1$$

$$T_2 = S_1 T_1 = 1$$

$$T_3 = S_2 T_2 = 0.9$$

$$T_4 = S_3 T_3 = 0.9$$

In this case $T = \sum_{i=1}^{4} n_i T_i = (2)(1) + (1)(1) + (3)(0.9) + (2)(0.9) = 7.5$

With $C(x) = \frac{1}{T} \sum_{i=1}^{4} A_i T_i$ where $A_i = \sum_{j=1}^{n_i} C_{ij}(x)$ we have

$$C(x) = \frac{1}{7.5}((1)(1.7) + (1)(.9) + (0.9)(2) + (0.9)(1.9) = \frac{1}{7.5}(6.11) = 0.815$$

Another approach for calculating the S_i involves associating with each criteria in H_i an additional local weight. In this case our form for H_i is

$$H_i = \{(C_{ij}, g_{ij}) \mid j = 1 \text{ to } n_i \}$$

where the g_{ij} indicates the importance of C_{ij} in calculating S_i. Here we assume that $g_{ij} \in [0, 1]$ and $\sum_{j=1}^{n_i} g_{ij} = 1$. Using these weights we can calculate $S_i = \sum_{j=1}^{n_i} g_{ij} C_{ij}(x)$.

An interesting special case of this is where some criteria C_{ij} has $g_{ij} = 0$. In this case the criteria plays no role in the determination of S_i but still is able to contribute to the overall calculation of $C(x)$.

Another available method for calculating the S_i involves the idea of combining these local weights with a tolerance level. Here we assume for each H_i we have $H_i = \{(C_{ij}, g_{ij}), j = 1 \text{ to } n_i\}$, $g_{ij} \in [0, 1]$ and $\sum_{j=1}^{n_i} g_{ij} = 1$, where again g_{ij} is the indication at the importance of C_{ij} in calculating S_i. In addition we assume a tolerance level $\alpha_i \in [0, 1]$ associated with H_i. Using one of the methods for generating OWA weights we can obtain a set of OWA weights, V_{ik}, for $k = 1$ to n_i. Let nd_i be an index such $nd_i(k)$ is the index of the k largest of the $C_{ij}(x)$. That is $b_{ik} = C_{i,nd_i(k)}(x)$ is the value of the k most satisfied criteria in H_i. With $d_{ik} = g_{i,nd_i(k)}$ being the importance weight associated with this kth most satisfied criteria on H_i we calculate $h_{ik} = \dfrac{d_{ik} \cdot V_{ik}}{\sum\limits_{k=1}^{n_i} d_{ik} \cdot V_{ik}}$ Using this we

calculate $S_i = \sum\limits_{K=1}^{n_i} h_{ik} b_{ik}$. In the special case when $V_{ik} = \dfrac{1}{n_i}$ for all k this reduces to the weighted average introduced earlier, $S_i = \sum\limits_{j=1}^{n_i} g_{ij} \cdot C_{ij}(x)$.

4. Prioritized "and" Operator

In the following we shall consider a related aggregation method called prioritized *anding*. We refer to this as the PRI-AND aggregation operator.

We recall that the "and" operator is generalized by a t-norm [9, 10]. A t-norm is a mapping.

$$R: [0. 1] \times [0, 1] \rightarrow [0, 1]$$

having the properties

      1. Symmetry: $R(a, b) = R(b, a)$

      2. Monotonicity: If $a \geq c$ and $b \geq d$ then $R(a, b) \geq R(c, d)$

      3. Associativity: $R(a, R(b, c)) = R(R(a, b), c)$

      4. 1 as identity: $R(1, a) = a$

The associativity property allows us to extend this to any number of arguments. An interesting property of the t-norm is $R(a_1, ..., a_n) \geq R(a_1, ..., a_n, a_{n+1})$.

A large number of possible examples of t-norms exist [11]. Three of the most important are

$R_M(x, y) = Min(x, y)$ Minimum

$R_P(x, y) = xy$ Product

$R_L(x, y) = Max(x + y - 1, 0)$ Lukasiewicz

It can be shown that for any x, y, $R_M(x, y) \geq R_P(x,y) \leq R_L(x, y)$. It is also true that for any t–norm R it is the case that $R_M(x, y) \geq R(x, y)$

We now look at the issue of performing the t-norm aggregation when the arguments have importance weights associated with them [12, 13]. Consider the aggregation $R((a_1, w_1), (a_2, w_2), ..., (a_n, w_n))$ where $w_j \in [0, 1]$ is the importance weight associated with the argument a_j. In [14] Yager suggested that we can implement this aggregation as

$$R((a_1, w_1), ..., (a_n, w_n)) = R(a_i^{w_i}, ..., a_n^{w_n}).$$

For example in the case where $R = R_P$ we have

$$R((a_1, w_1), ...(a_n, w_n)) = \prod_{i=1}^{n} a_i^{w_i}.$$

In the case where $R = R_M$ then

$$R((a_1, w_1), ...(a_n, w_n)) = Min_i[a_i^{w_i}]$$

We note that if $w_i = 0$ then $a_i^0 = 1$. Since one is the identity of the t-norm then criteria with zero importance have no effect in the calculations of $R(a_i^{w_i}, ... a_n^{w_n})$.

In [15] Yager suggested alternate methods for implementing weighted t-norm aggregations. While we shall not discuss these here we do note that the methodology introduced in the following can be easily applied to any of the other methods for including importance.

In the following we introduce a prioritized 'and' operator. We refer to this as the PRI-AND aggregation operator.

Again we have a collection of criteria partitioned into q categories $\{H_1, ..., H_q\}$ such that $H_i = \{C_{i1}, ..., C_{in_i}\}$. Again we assume a prioritization of the categories, $H_1 > H_2 ....> H_q$. Our objective is to obtain a prioritized 'anding' aggregation of the satisfaction of these criteria by some alternative x. We assume $C_{ij}(x) \in [0, 1]$ is the satisfaction of criteria C_{ij} by alternative x.

We first calculate for each category $S_i = Min_j[C_{ij}(x)]$. We next calculate

$$T_i = \prod_{k=1}^{i} S_{k-1}$$

with the understanding that $S_O = 1$ by definition. We now define the prioritized weight associated with C_{ij} as $w_{ij} = T_i$. We now calculate the PRI-AND aggregation of the $C_{ij}(x)$ using the t--norm R as

$$C(x) = \mathop{\mathbf{R}}_{i,\,j} [(C_{ij}(x))^{w_{ij}}]$$

Since $w_{ij} = T_i$ then $C(x) = \mathop{\mathbf{R}}_{i} \mathop{\mathbf{R}}_{j} [(C_{ij}(x))^{T_i}]$

In the case where R is the Min then we get $C(x) = \text{Min}_i[\text{Min}_j(C_{ij(x)}^{T_i})]$

Noting that $\text{Min}_j(C_{ij}(x)) = S_i$ we have

$$C(x) = \text{Min}_i[S_i^{T_i}]$$

In the case where R is the product t-norm we have

$$C(x) = \prod_i \prod_j [(C_{ij}(x))^{T_i}]$$

If we get $D_i(x) = \prod_{j=1}^{n_i} C_{ij}(x)$ then we get

$$C(x) = \prod_{l=1}^{q} (D_i)^{T_i}$$

If we take the log of the above we have

$$\text{Log}(C(x)) = \sum_{i=1}^{q} T_i\, \text{Log}(D_i)$$

Since $D_i(x) = \prod_{j=1}^{n_i} C_{ij}(x)$ then $\text{Log}(D_i) = \sum_{j=1}^{n_i} \text{Log}(C_{ij}(x))$. Hence in this case we have

$$\text{Log}(C(x)) = \sum_{i=1}^{q} T_i\, \text{Log}(D_i) \ \text{ and } \ \text{Log}(D_i) = \sum_{j=1}^{n_i} \text{Log}(C_{ij}(x))$$

This form looks very similar to the weighted average where

$$C(x) = \frac{1}{T} \sum_{i=1}^{q} T_i A_i \ \text{ and } \ A_i = \sum_{j=1}^{n_i} C_{ij}(x).$$

5. Prioritized "or" Operator

We now consider a related aggregation method the prioritized *anding*. We refer to this as

PRI-OR aggregation operator.

We recall that the "or" operator is generalized by a t-conorm [9], a mapping.

$$P: [0. 1] \times [0, 1] \rightarrow [0, 1]$$

having the properties

1. Symmetry: $P(a, b) = P(b, a)$

2. Monotonicity: If $a \geq c$ and $b \geq d$ then $P(a, b) \geq P(c, d)$

3. Associativity: $P(a, P(b, c)) = P(P(a, b), c)$

4. 0 as identity: $P(0, a) = a$

A property of the t-conorm is $P(a_1, ..., a_n) \leq P(a_1, ..., a_n, a_{n+1})$.

Three important are examples of this are

$P_M(x, y) = Max(x, y)$ Maximum

$P_S(x, y) = x + y - xy$ Probabilistic Sum

$P_L(x, y) = Min(x + y, 1)$ Lukasiewicz

It is well know that for any t–conorm P it is the case that $P_M(x, y) \leq P(x, y)$, max is the smallest.

We now look at the issue of performing the t-conorm aggregation when the arguments have importance weights associated with them. Consider the aggregation $P((a_1, w_1), (a_2, w_2), ..., (a_n, w_n))$ where $w_j \in [0, 1]$ is the importance weight associated with the argument a_j. In [12] Yager suggested that we can implement this aggregation as

$$P((a_1, w_1), ..., (a_n, w_n)) = P(w_1 a_1, w_2 a_2, ..., w_n a_n).$$

We aggregate the product of w_j times a_j. For example in the case where $P = P_M$ we have

$$P_M((a_1, w_1), ..., (a_n, w_n)) = Max_i[w_i a_i]$$

Consider now the case of probabilistic sum. Since $R_p(x. y) = 1 - (1- x)(1 - y)$ then

$$P_s(a_1, w_1), ..., (a_n, w_n)) = 1 - \prod_{i=1}^{n} (1 - w_i a_i)$$

We note that if $w_i = 0$ then $w_i a_i = 0$. Since zero is the identity of the t-conorm then criteria with zero importance have no effect in the calculations of $P(w_1 a_1, w_2 a_2, ..., w_n a_n)$.

In the following we introduce a prioritized 'or' operator, the PRI-OR aggregation operator.

Here we have a collection of criteria partitioned into q categories $\{H_1, ..., H_q\}$ such that $H_i =$

$\{C_{i1}, ..., C_{in_i}\}$. Again we assume a prioritization of the categories, $H_1 > H_2 .... > H_q$. Our objective is to obtain a prioritized 'oring' aggregation of the satisfaction of these criteria by some alternative x. We assume $C_{ij}(x) \in [0, 1]$ is the satisfaction of criteria C_{ij} by alternative x.

We first calculate for each category $S_i = Max_j[C_{ij}(x)]$. We next calculate

$$T_i = \prod_{k=1}^{i} S_{k-1}$$

with the understanding that $S_O = 1$ by definition. We define the prioritized weight associated with C_{ij} as $w_{ij} = T_i$. We now calculate the pri-or aggregation of the $C_{ij}(x)$ using the t--conorm P as

$$C(x) = \mathbf{P}_{i,j} [(w_{ij} C_{ij}(x))]$$

Since $w_{ij} = T_i$ then $C(x) = \mathbf{P}_i \mathbf{P}_j [(T_i C_{ij}(x))$

To get a feel for this we consider the special case where each category has just one element, $H_i = \{C_i\}$ and P is the probabilistic sum. In this case $S_i = C_i(x)$ with $S_O = 1$. Furthermore

$$T_i = \prod_{k=1}^{i} S_{k-1} = \prod_{k=1}^{i-1} C_k(x).$$ In this case

$$C(x) = 1 - \prod_{i=1}^{q} (1 - (C_i(x) \prod_{k=1}^{i-1} C_k(x))) = 1 - \prod_{i=1}^{q} (1 - \prod_{k=1}^{i} C_k(x))$$

For the case where q = 2 we have

$$C(x) = 1 - (1 - C_1(x)) (1 - C_1(x)C_2(x)) = C_1(x) + C_1(x) C_2(x) - C_1(x)C_1(x) C_2(x)$$

$$C(x) = C_1(x)(1 + C_2(x)(1 C_1(x)) = C_1(x)(1 + C_2(x) \overline{C_1(x)})$$

Thus if $C_1(x) = 1$ then $C(x) = 1$, $C_1(x) = 0$ then $C(x) = 1$. If for example $C_1(x) = 0.7$ then

$$C(x) = 0.7(1 + 0.3 C_2(x))$$

On the other hand if $C_2(x) = 0$ then $C(x) = C_1(x)$ while if $C_2(x) = 1$ then

$C(x) = C_1(x) + C_1(x) \overline{C_1(x)}$.

6. Conclusion

We considered criteria aggregation problems where there is a prioritization relationship over the criteria. We suggested that prioritization between criteria can be modeled by making the weights associated with a criteria dependent upon the satisfaction of the higher priority criteria. This resulted

in a situation in which the weights associated with the criteria depended upon the alternative being evaluated. We introduec a number of aggregation operators in which there exists a prioritization relationship between the arguments. We first introduced a prioritized averaging operator. We next introduced a prioritized "anding" and then a prioritized "oring" operator.

7. References

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